

Online Appendix
for
A Hedge Fund in Your Garage:
Automobile purchases under gasoline price uncertainty*

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Preliminary and incomplete

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APPENDIX A: PROOFS OF CONSUMER VEHICLE CHOICE MODEL RESULTS

This appendix provides proofs of results derived from the vehicle choice model presented in Section ???. This Appendix is intended as a supplement to the manuscript. Refer to Section ??? for full details of the model.

A.1 Background and Definitions

In each period t the consumer observes gasoline prices f_t . They derive utility $h(V)$ from V units of VMT with $h' < 0$ and $h'' < 0$. Given their endowment ω_t , utility from vehicle i 's attributes $A(x_i)$, and vehicle fuel intensity g_i , they optimally chose VMT in that period to maximize currently period utility. Repeating Equation 4 this is

$$V_t(f_t|i = i) = \underset{V}{\operatorname{argmax}} \{ \omega_t + A(x_i) + h(V) - f_t g_i V \} \quad (\text{A.1})$$

Following the consumer vehicle choice model presented in Section ???, define the marginal willingness to pay for fuel intensity ($MWTPg$) as the derivative of willingness to pay for a vehicle with respect to the fuel intensity of that vehicle. Repeating Equation 5 this can be derived as

$$MWTPg = \frac{1}{R'_0} \sum_{t=1}^T \left[\beta^t \int -f_t V(f_t \cdot g_i) R'(\omega_t + A(x_i) + h(V(f_t \cdot g_i)) - V(f_t \cdot g_i) \cdot f_t g_i) \Omega_t^f(f_t) df_t \right] \quad (\text{A.2})$$

For notational simplicity we define the following:

Definition A.1. Let $R'_t(\Delta f_t)$ be the marginal utility of consumption in period t under a gasoline price change of Δf_t from period 0, or

$$R'_t(\Delta f_t) = R'(\omega_t + A(x_i) + h(V((f_0 + \Delta f_t) \cdot g_i)) - V((f_0 + \Delta f_t) \cdot g_i) \cdot (f_0 + \Delta f_t) g_i)$$

A.2 Propositions

Proposition A.1. *There is a unique $V(f_t) : f_t \rightarrow V_t$ that maximizes utility in period t*

Proof. Consider Equation A.1. Assuming the consumer is not at a corner solution of $V = 0$, then utility maximization requires $h'(V) = f_t g_i$. $h' < 0$ and $h'' < 0$ so $\exists! V$ such that $h'(V) = f_t g_i$ and therefore a unique V that maximizes utility. \square

Corollary A.1. *In each period the consumer values the marginal VMT at the operating cost, or $h'(V_t) = f_t g_i$*

Proof. Following from Corollary A.1, if V_t maximizes current period utility it must be that the $h'(V_t) = f_t g_i$. \square

Corollary A.2. *The consumer's VMT is decreasing in increasing gasoline prices or $\frac{\partial V_t}{\partial f_t} < 0$*

Proof. Following from Corollary A.1, at the utility maximizing choice of V_t , $h'(V) = f_t g_i$. Given $h'' < 0$, by the Implicit Function Theorem, $\frac{\partial V_t}{\partial f_t} < 0$. \square

Proposition A.2. *The consumer's total utility is decreasing in increasing fuel prices*

Proof. From Corollary A.2, VMT is decreasing in increasing fuel prices. By Corollary A.2, at some optimal VMT V_t the marginal utility of VMT is the marginal vehicle operating cost $f_t g_i$. Consider increasing fuel prices in period t to $\hat{f}_t > f_t$. Since the outside good is numeraire, utility in period t must decrease. Utility in other periods does not depend on f_t , thus overall utility must decrease. \square

Proposition A.3. *The function $\frac{R'_t(\Delta f_t)}{R'_0}$ is decreasing in Δf_t*

Proof. By definition $R'(u) > 0$ and $\Delta f_t = f_t - f_0$. Holding f_0 fixed, increasing Δf_t requires increasing f_t . By Proposition A.3, this means utility in period t decreases and, therefore \mathbf{R} must decrease as well. \square

A.3 Results

Result A.1. *Marginal willingness to pay for fuel intensity under risk neutrality*

Suppose the consumer is risk neutral. Then $R'(u) = \mathbf{R}' \forall u$. From Equation A.2

$$\begin{aligned} MWTPg &= \frac{1}{\mathbf{R}'} \sum_{t=1}^T \left[\beta^t \int -f_t V(f_t \cdot g_i) \mathbf{R}'(-V(f_t \cdot g_i) \cdot f_t g_i) \Omega_t^f(f_t) df_t \right] \\ MWTPg &= - \sum_{t=1}^T \beta^t \int [f_t V(f_t g_i)] \Omega_t^f(f_t) df_t \quad \blacksquare \end{aligned}$$

Result A.2. *Marginal willingness to pay for fuel intensity under risk preferences with certainty*

Then from Equation A.2, and substituting \mathbf{R}' from Definition A.1

$$MWTPg = \frac{1}{R'_0} \sum_{t=1}^T \left[\beta^t \int -f_t V(f_t \cdot g_i) \frac{R'_t(\Delta f_t)}{R'_0} \Omega_t^f(f_t) df_t \right]$$

Under the assumption of no uncertainty, realizations of fuel prices are f_t with probability one. Thus, Ω_t^f is the Dirac delta function centered on f_t , or

$$\Omega_t^f(f) = \begin{cases} \infty & f = f_t \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \int \Omega_t^f(f) df = 1$$

Evaluating the integral we obtains

$$MWTPg = - \sum_{t=1}^T \beta^t [f_t V(f_t g_i)] \frac{R'_t(\Delta f_t)}{R'_0} \quad \blacksquare$$

Result A.3. *Marginal willingness to pay for fuel intensity under risk preferences with uncertainty*

Consider the first-order Taylor approximation of $MWTPg$ around $\sigma = 0$.

$$MWTPg = \left. \frac{\partial EU}{\partial g} \right|_{\sigma=0} + \sigma \left. \frac{\partial^2 EU}{\partial g \partial \sigma} \right|_{\sigma=0} + \dots$$

Since $\sigma = 0$ implies no uncertainty, the first term in this expansion is $MWTPg$ with risk aversion under certainty and is derived as Proposition A.2. The second term is the derivative of this term with respect to σ or

$$\begin{aligned} \frac{\partial^2 EU}{\partial g \partial \sigma} &= \frac{\partial \partial}{\partial \sigma} \left[- \sum_{t=1}^T \beta^t [f_t V(f_t g_i)] \frac{R'_t(\Delta f_t)}{R'_0} \right] \\ &= \frac{\partial}{\partial \sigma} \left[\frac{1}{R'_0} \sum_{t=1}^T \left[\beta^t \int -f_t V(f_t \cdot g_i) \frac{R'_t(\Delta f_t)}{R'_0} \Omega_t^f(f_t) df_t \right] \right] \\ &= \sum_{t=1}^T \beta^t \int \left[f_t V(f_t g_i) \frac{R'_t(\Delta f_t)}{R'_0} \frac{\partial \Omega_t^f}{\partial \sigma} \Big|_{\sigma=0} \right] df_t \end{aligned}$$

This gives the following expression for the first-order Taylor expansion

$$MWTPg \approx - \sum_{t=1}^T \beta^t [f_t V(f_t g_i)] \frac{R'_t(\Delta f_t)}{R'_0} + \sigma \sum_{t=1}^T \beta^t \int \left[f_t V(f_t g_i) \frac{R'_t(\Delta f_t)}{R'_0} \frac{\partial \Omega_t^f}{\partial \sigma} \Big|_{\sigma=0} \right] df_t$$

APPENDIX B: UNCERTAINTY IN FUTURE FUEL PRICES

B.1 Uncertainty in Future Fuel Prices

Using NYMEX data on commodity futures and options prices, we compute the time series of implied volatility for RBOB gasoline and West Texas Intermediate crude oil (WTI) from the earliest available date of options data to the start of 2023 applying the Black-Scholes-Merton (BSM) asset pricing model as described in [Section 4.1.2](#). This section provides descriptive on these volatility series and motivates our choice of WTI implied volatility as the preferred measure of future price uncertainty.

B.1.1 Crude Oil Volatility as a Proxy for Gasoline Volatility

In the bulk of our analysis to follow, we will use implied volatility derived from crude oil markets, specifically WTI, as a proxy for volatility in retail gasoline prices. We prefer this measure due to two data limitations. First, options data for refined gasoline are available starting in 2007, whereas our data provider reports WTI options data as far back as the start of 2005.

Second, there is substantially more trade in crude oil options compared to refined gasoline options. For some days there is little to no trade in refined gasoline options contracts, preventing calculation of volatility on those dates. Further, thickly-traded options are likely to lead to more accurate measures of implied volatility. [Figure B.1](#) shows a time series of log scale 4-week total trading volumes for futures (Panel (a)) and options (Panel (b)) for WTI and Reformulated Blendstock for Oxygenated Blending (RBOB). There is consistently substantially larger trading volume for WTI contracts, particularly in options where the typical volume for crude oil futures is two orders of magnitude larger than refined gasoline contract volume.

[Figure 1 about here.]

Implied volatility from these two commodities are strongly correlated over time and the WTI series approximates the RBOB series with very little error. [Figure B.2](#) compares implied volatility for a refined gasoline product (RBOB) and crude oil (WTI) over the period where data are available.¹ Panel (a) directly compares the monthly average implied volatility of WTI and RBOB derived from 2-month forward options contracts. The time series of these patterns are broadly similar, however RBOB volatility averages somewhat larger in levels.² The correlation of these two series is 0.816.

[Figure 2 about here.]

To further investigate this relationship, we estimate a regression of RBOB volatility on WTI volatility. Panel (b) shows a time series plot comparing implied volatility of RBOB (black) against the predicted RBOB volatility this regression. The red line shows the predicted RBOB volatility from in-sample forecasts using a single regression. A potential concern is that in-sample predictions are overfit to idiosyncratic factors and do not represent the true relationship between RBOB and WTI volatility. As an alternative, we estimate this same regression, fitting a model to rolling 26-week windows of data and using that model to predict RBOB volatility using realized WTI volatility four weeks after the end of the training window. The results of these forecasts are shown using the blue line. Both the in- and out-of-sample forecasts closely follow the true RBOB volatility series with correlations of 0.816 and 0.817, respectively.

An additional concern may be that general macroeconomic uncertainty is the primary determinant of oil price uncertainty.³ In analysis provided in the Appendix, we demonstrate that there is substantial independent variation in the level of oil prices, implied volatility in oil prices, and measures of general macroeconomic uncertainty. Further, we perform robustness

¹RBOB options data are available starting in mid-2006 and trading has been sparse since 2021.

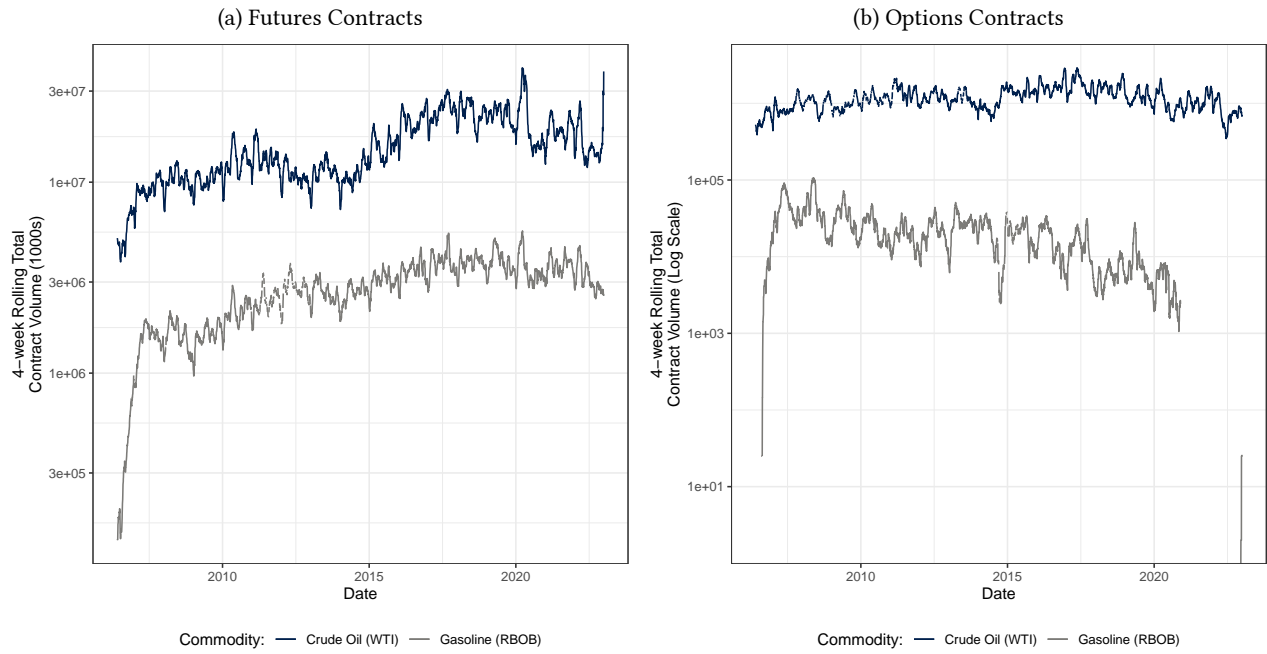
²Implied volatility is the variance in log returns and thus is unitless and invariant to the scale of prices in the underlying commodity. From this we conclude there is more uncertainty in future RBOB prices than would be predicted from the WTI uncertainty alone.

³*e.g.*, [BaumeisterEtAl:2013:OilShocks](#) show oil price shocks have a modest impact on the US economy that has declined since the mid-1980s. More recently, [Killian:2009:OilPriceShocks](#) finds oil price shocks flow through into macroeconomic shocks, with supply shocks having a larger and more persistent impact than demand shocks.

analysis where we exclude periods after large macroeconomic shocks: *i.e.*, the 2008 financial crisis (December 2007 to June 2009), the initial COVID-19 pandemic lockdown periods (March 2020 to May 2021) and the start of the Russian invasion of Ukraine (February 2022 to January 2023).⁴

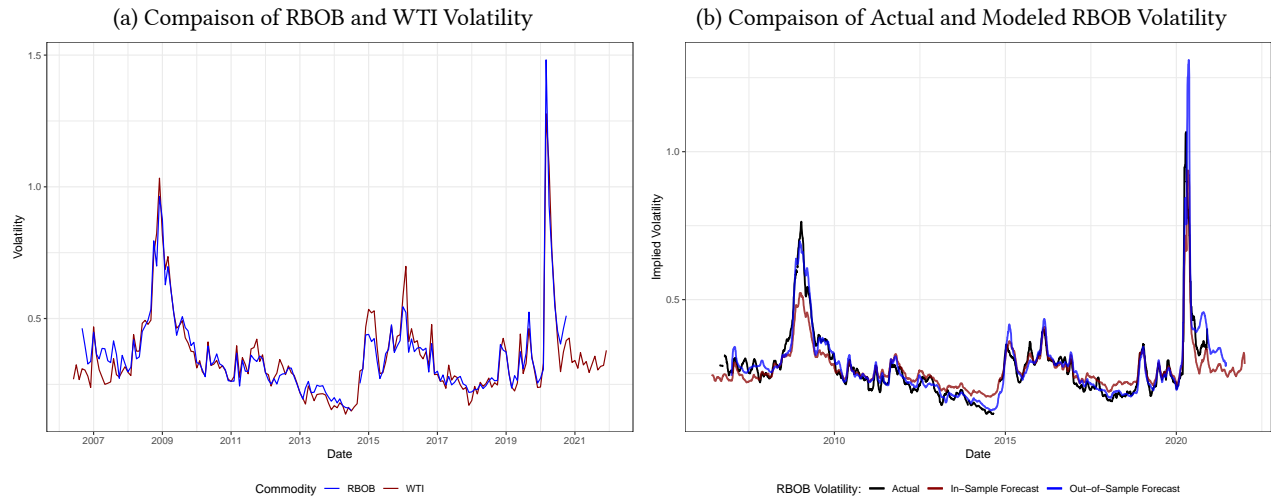
⁴Due to data availability, our analysis of automobile purchases ends prior to the start of the COVID-19 pandemic and the final two periods are excluded in all estimates. If we obtain additional automobile sales data prior to 2010 and data on WTI volatility prior to 2005 we will also similarly exclude for robustness the post-9/11 recession (September 2001 to August 2002) and the Asian Financial Crisis (July 1997 to December 1998).

Figure B.1: Comparison of Refined Gasoline and Crude Oil Contract Volume



Comparison of daily NYMEX futures (panel (a)) and options (panel (b)) contract volumes refined gasoline (RBOB) and crude oil (WTI). Due to the large scale difference in options contract volumes, the vertical axes use a log scale.

Figure B.2: Crude Oil Volatility is Strongly Correlated with Refined Gasoline Volatility



Panel (a) directly compares the monthly average implied volatility of WTI and RBOB derived from 2-month forward options contracts. Panel (b) compares implied volatility of RBOB computed using 2-month forward options (black) against predicted RBOB volatility from a regression of RBOB on WTI volatility. The red line shows in-sample forecasts from a single regression. The blue line shows 4-week ahead (out-of-sample) forecasts from a model trained on a rolling 26-week sample.

APPENDIX C: ADDITIONAL EMPIRICAL RESULTS

C.1 Additional Implied Volatility Descriptives

In this section we provide additional empirical descriptives of our implied volatility measure.

C.1.1 Prices and Implied Volatility Vary Independently

We do observe correlation in WTI price and implied volatility. Here we characterize the variation in these series. In [Figure C.3](#) we plot the time series of WTI front-month futures price, implied volatility, and residuals from a regression of implied volatility on prices. Each series is presented as a z-score. While there is weak correlation between the series with a regression of volatility on forward prices having an R^2 of 0.108. Oil prices do little to explain future uncertainty and the residual variation in implied volatility after conditioning out future prices closely follows the implied volatility series.

[Figure 3 about here.]

C.1.2 Oil Price Volatility and Macroeconomic Uncertainty

There is a substantial literature suggesting a connection between oil price shocks and broader macroeconomic shocks.⁵ It then stands to reason one may be concerned that WTI volatilities may only be capturing general macroeconomic uncertainty. To address this issue, we compare WTI volatilities to general macroeconomic uncertainty measured by CBOE VIX Index (VIX).

There is some correlation in these series; a regression of WTI volatility on VIX has an R^2 of 0.328. We plot this relationship in [Figure C.4](#). Panel (a) shows a time series plot of VIX (blue) and WTI volatility (red). Each series is standardized to a z-score. Panel (b) shows the residual from a regression of WTI volatility on VIX. From both figures it is clear there is still substantial independent residual variation in the WTI volatility series conditional on VIX. While this analysis shows general macroeconomic uncertainty has only weak explanatory power over WTI volatility, we include VIX as a control in any analysis where we do not directly control for time-varying confounders using fixed effects.

[Figure 4 about here.]

It is also clear from [Figure C.4](#) there are periods of very large in magnitude oil price and macroeconomic uncertainty. To ensure our later results are not solely driven by these periods of extreme uncertainty, we perform robustness analysis where we exclude the 2008 financial crisis (December 2007 to June 2009), the initial COVID-19 pandemic lockdown period (March 2020 to May 2021) and the start of the Russian invasion of Ukraine (February 2022 to January 2023).⁶ These periods are shaded in gray.

C.2 Additional Results on the Willingness to Pay for Fuel Economy **XXX[Jim]**

This section includes additional results on the impact of future price uncertainty on the willingness-to-pay (WTP) for fuel economy.

[Table C.1](#) repeats the simulation in [Table 5](#), but increases the time series of implied volatility by one standard deviation.

[Table 1 about here.]

[Figure C.5](#) repeats [Figure 1](#), adding shaded regions for standard errors computed using 10,000 bootstrap iterations.

[Figure 5 about here.]

⁵e.g., [BaumeisterEtAl:2013:OilShocks](#) show oil price shocks have a modest impact on the US economy that has declined since the mid-1980s. More recently, [Killian:2009:OilPriceShocks](#) finds oil price shocks flow through into macroeconomic shocks, with supply shocks having a larger and more persistent impact than demand shocks.

⁶Do do data availability, our analysis of automobile purchases ends prior to the start of the COVID-19 pandemic and the final two periods are excluded in all estimates. If we obtain additional automobile sales data prior to 2010 and data on WTI volatility prior to 2005 we will also similarly exclude for robustness the post-9/11 recession (September 2001 to August 2002) and the Asian Financial Crisis (July 1997 to December 1998).

C.3 Prices vs. Quantities in Climate Regulation

C.4 Counterfactual Fuel Economy

The counterfactual simulation changes not only the mean but also the variance of fuel economy in vehicle purchased. [Table C.2](#) computes the standard deviation of fuel economy in MPG for each vehicle class under each counterfactual. Variance in fuel economy decreases across all vehicle classes under the price instrument, reflecting sales aggregating around vehicles with improved fuel economy. This is not uniformly the case under the quantity instrument, however, with variance increasing in some classes and decreasing in others.

[Table 2 about here.]

C.4.1 Counterfactual Emissions

We perform a back-of-the envelope calculation of the lifetime vehicle emissions reductions implied by the price (PI) and quantity (QI) instruments for GHG abatement. Using our preferred demand specification, we calculate sales in each quarter under a baseline scenario and two counterfactual scenarios. The baseline scenario uses observed gasoline prices and implied volatility, while the counterfactual scenarios use prices and implied volatility calculated under PI and QI. Each counterfactual predicts a change in vehicle sales relative to the baseline consisting of a shift in the shares of vehicle sold and a shift in consumers electing the outside good, which we assume is electing not to purchase a new vehicle and instead drive their existing vehicle. We then compute the lifetime emissions for all vehicles sold under each scenario under the following assumptions:

1. Vehicles are driven 15,000 miles per year in baseline
2. Vehicles last for 16.5 years and are then scrapped
3. Consumers who elect to consume the outside good drive a vehicle with a MPG rating equal to the mean MPG rating of all vehicles sold in that year in the baseline scenario⁷
4. We consider two possible responses of consumers to operating costs of their vehicle
 - (a) **No VMT Response:** The operating cost-VMT elasticity is zero and each vehicle is driven 15,000 miles per year regardless of fuel prices and fuel efficiency
 - (b) **/w VMT Response:** The operating cost-VMT elasticity is -0.45 and vehicles will be driven less due to higher counterfactual fuel prices but more due to the shift toward more fuel efficient vehicles in each counterfactual.

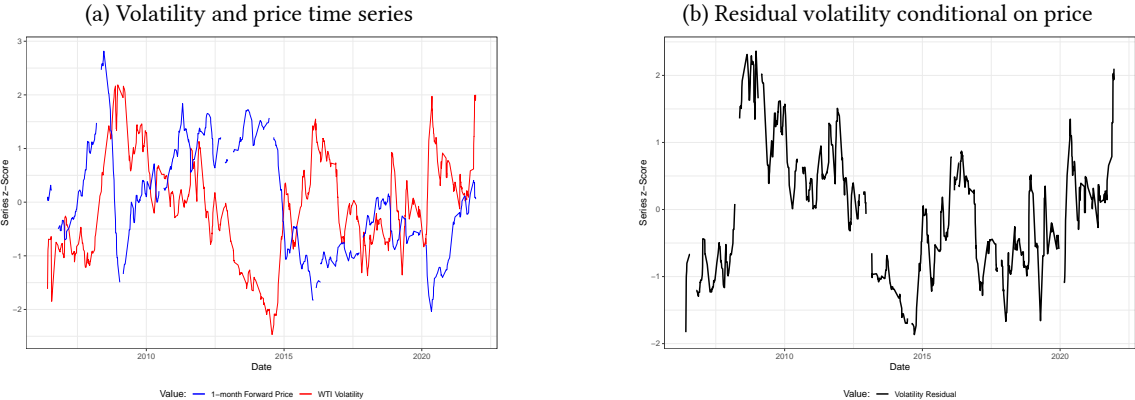
We then compute the percent change in CO₂ emissions over the life of all vehicles sold in that year relative to the baseline scenario. The results are shown in [Table C.3](#). There are two important takeaways. First, emissions reductions are substantially larger when driver's VMT decision is price responsive.

Next, when there is no VMT response, the QI always leads to larger emissions reductions than the PI. Consumers on average purchase more efficient vehicles under the QI, which will lead to lower emissions if they do not change driving behavior. When the VMT response is non-zero, however, emissions reductions under the QI and PI are quite similar and, in some cases, higher under the QI. This arises from the fact that under the QI, the average vehicle is more efficient than under the PI and drivers are less exposed to higher prices from the GHG policy and consequently drive more than under the PI. Additionally, new vehicle sales are substantially lower under the QI, leaving less efficient vehicles driving longer.

[Table 3 about here.]

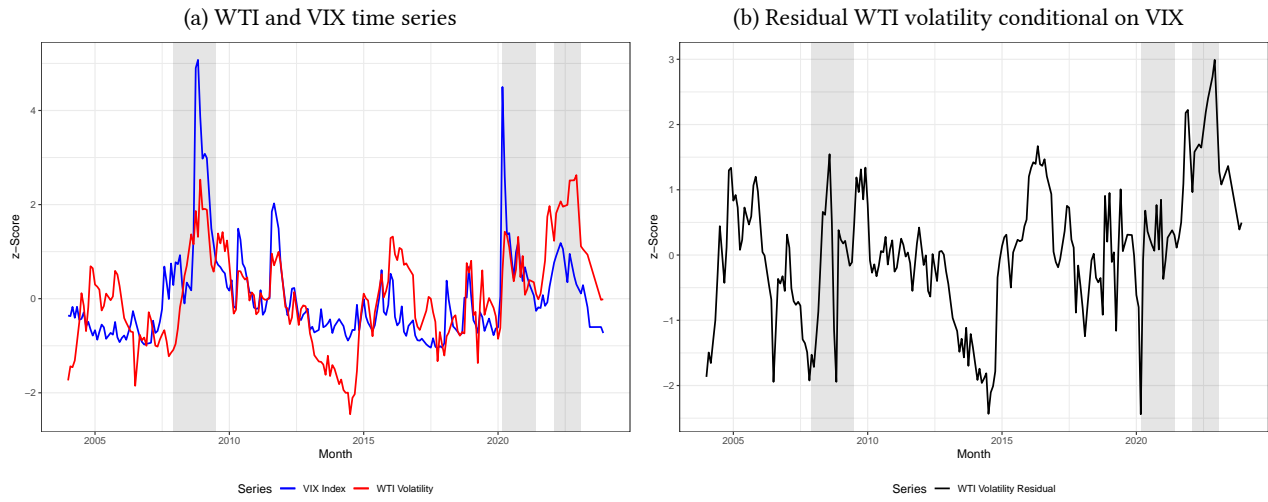
⁷New vehicles tend to be more fuel efficient than the vehicle they replace and this assumption likely understates the true fuel consumption of the outside vehicle.

Figure C.3: Time Series of WTI Crude Oil Implied Volatility and Forward Prices



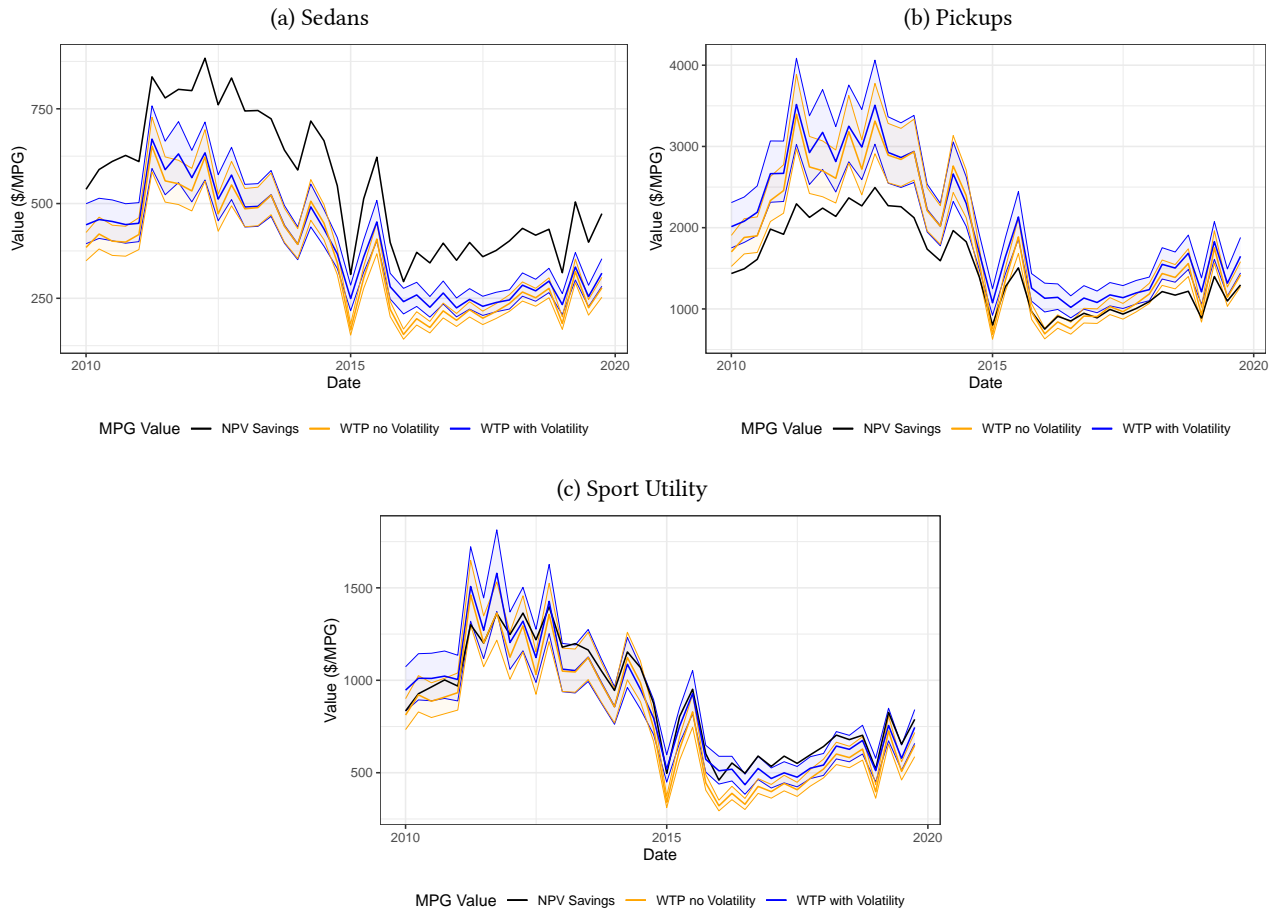
Panel (a) is a time series plot of 4-week moving averages of the 1-month forward price, 12-month implied volatility. Panel (b) plots the residual variation in WTI volatility conditional on the forward oil price. All series converted to z-scores for a consistent scale.

Figure C.4: Time Series of Crude Oil Implied Volatility and Macroeconomic Uncertainty



Panel (a) is a time series plot of the implied volatility from 1-year forward WTI options and the CBOE VIX index, which measures general macroeconomic uncertainty. Panel (b) plots the residual variation in WTI volatility conditional on VIX. All series converted to z-scores for a consistent scale.

Figure C.5: Simulated WTP for a 1 MPG Improvement in Fuel Efficiency



Simulation of the WTP for a one dollar reduction in present discounted lifetime operating costs for the sales-weighted median price and fuel efficiency vehicle, using historical gasoline prices and implied volatility. Panels (a), (b), and (c) show sedans, pickups, and SUVs, respectively. Vehicles are assumed to be driven 15,000 miles per year and last 16.5 years. The blue line performs the calculation using a model excluding the uncertainty effect (Column 1 of [XXXtable](#)). The orange line uses our preferred model, including the volatility effect (Column 2 of [XXXtable](#)). Standard errors computed using 10,000 bootstrap iterations.

Table C.1: Impact of 1-sd Increase in Volatility on WTP Fuel Economy

	MWTP (\$/MPG)			
	Sedan	Pickups	Sport Utility	All Vehicles
NPV	\$553.70 [\$397.44 – \$719.28]	\$1,518.33 [\$1,001.83 – \$2,017.61]	\$879.08 [\$601.24 – \$1,155.94]	\$848.48 [\$558.30 – \$1,072.83]
WTP no Vol	\$357.27 [\$220.58 – \$476.26]	\$1,835.43 [\$1,055.53 – \$2,630.89]	\$781.08 [\$463.83 – \$1,034.97]	\$811.84 [\$384.06 – \$1,055.78]
WTP /w Vol	\$424.50 [\$286.09 – \$530.22]	\$2,229.20 [\$1,382.73 – \$3,006.42]	\$936.91 [\$598.78 – \$1,148.55]	\$977.49 [\$475.47 – \$1,280.62]
WTP no Vol to NPV	-35.48% [-44.50% – -33.79%]	20.88% [5.36% – 30.40%]	-11.15% [-22.85% – -10.46%]	-4.32% [-31.21% – -1.59%]
WTP /w Vol to NPV	-23.33% [-28.02% – -26.29%]	46.82% [38.02% – 49.01%]	6.58% [-0.41% – -0.64%]	15.20% [-14.84% – 19.37%]
WTP incr /w Vol	18.82% [11.33% – 29.70%]	21.45% [14.27% – 31.00%]	19.95% [10.97% – 29.10%]	20.40% [21.30% – 23.80%]

Comparison of the present value savings under a counterfactual 1-sd increase in implied volatility from marginal 1 MPG improvement in fuel economy against the WTP for that same improvement in fuel economy from the demand model in **XXXXTable ??**. For each vehicle class, we compute the quarterly median price and fuel economy and match to mean fuel prices and implied volatility for that quarter. NPVs are computed assuming vehicles have a lifetime of 16.5 years, are driven 15,000 miles per year, and consumers discount future costs at a rate identical to the 10-year US Treasury bond rate at the time of purchase. NPVs and WTPs vary over time. The mean value is shown in the first row of each group, with the interquartile range in brackets below. The final three rows compare undervaluation of MPG not accounting for volatility, undervaluation accounting for volatility, and the percentage increase in the willingness to pay when accounting for volatility.

Table C.2: Weighted Standard Deviation of Fuel Economy

Nest	Baseline MPG (1)	Weighted S.D. MPG		
		Baseline (2)	PI (3)	QI (4)
All Vehicles	25.2	0.804	0.760	0.757
Convertible	24.1	1.161	1.148	1.156
Coupe	24.0	0.594	0.577	0.602
Hatchback	36.6	1.354	1.282	1.320
Passenger Vans	21.6	0.983	1.010	1.057
Pickups	17.9	1.311	1.284	1.260
Sedan	29.1	1.448	1.431	1.444
Sport Utility	23.4	1.153	1.136	1.131
Station Wagon	29.4	1.717	1.711	1.756
Van	23.2	1.346	1.293	1.249

This table contains sales-weighted standard deviations of fuel economy for the baseline and counterfactual scenarios.

Table C.3: Percentage Change in Sales and Fuel Economy

Year	Change in CO2 Emissions			
	No VMT Response		/w VMT Response	
	PI	QI	PI	QI
2010	0.8%	1.2%	3.6%	3.6%
2011	0.8%	1.2%	2.7%	2.8%
2012	0.4%	1.3%	1.5%	1.8%
2013	0.3%	1.0%	1.0%	1.3%
2014	0.3%	0.7%	1.3%	1.5%
2015	0.3%	0.7%	2.2%	2.2%
2016	0.2%	0.7%	1.8%	1.9%
2017	0.2%	0.8%	1.7%	1.8%
2018	0.6%	1.0%	3.9%	3.8%
2019	0.8%	1.2%	6.0%	5.7%

The counterfactual percentage change in lifetime CO₂ emissions for all vehicles sold in the listed year under the price instrument (PI) and quantity instrument (QI). In the "No VMT Response" columns, vehicle owners drive 15,000 miles per year regardless of gasoline prices or vehicle fuel efficiency. In the "/w VMT Response" columns vehicles owners drive 15,000 miles per year in the baseline but have a -0.45 VMT-operating cost elasticity accounting for both higher gasoline prices and changes in selected vehicle fuel economy under the PI and QI.